Pattern Formation in Confined Plasmas: Why Staircases are Inevitable in Drift-Rossby Turbulence

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- Guilhem Dif-Pradalier; CEA, Cadarache

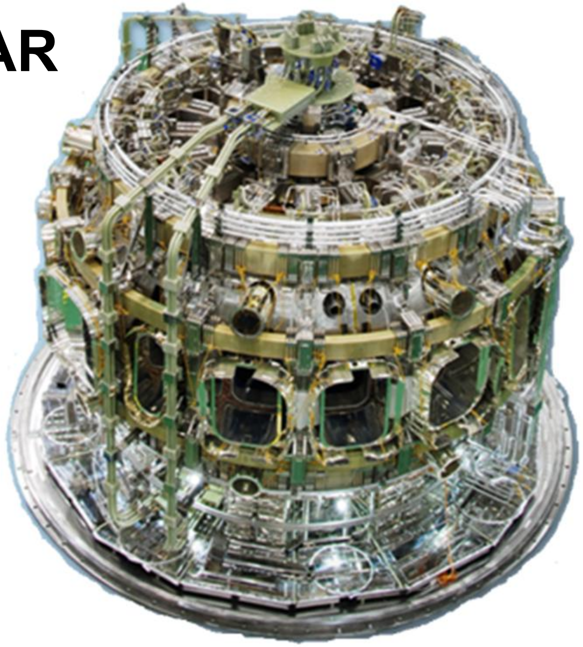
Contents

What is a Tokamak?

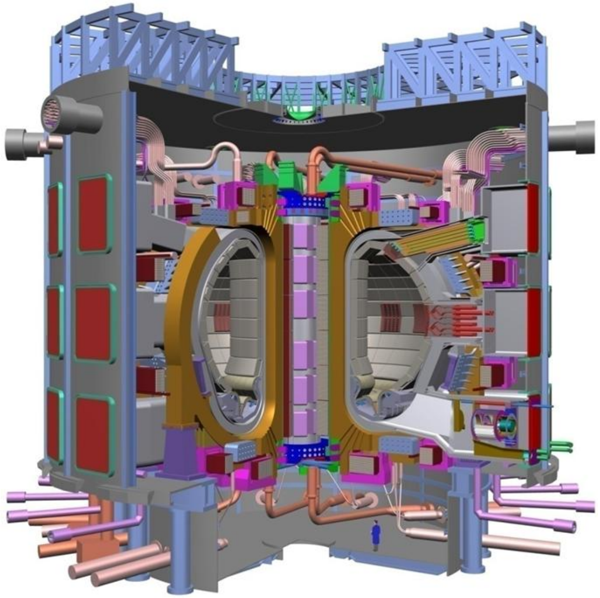
N.B. No advertising intended…

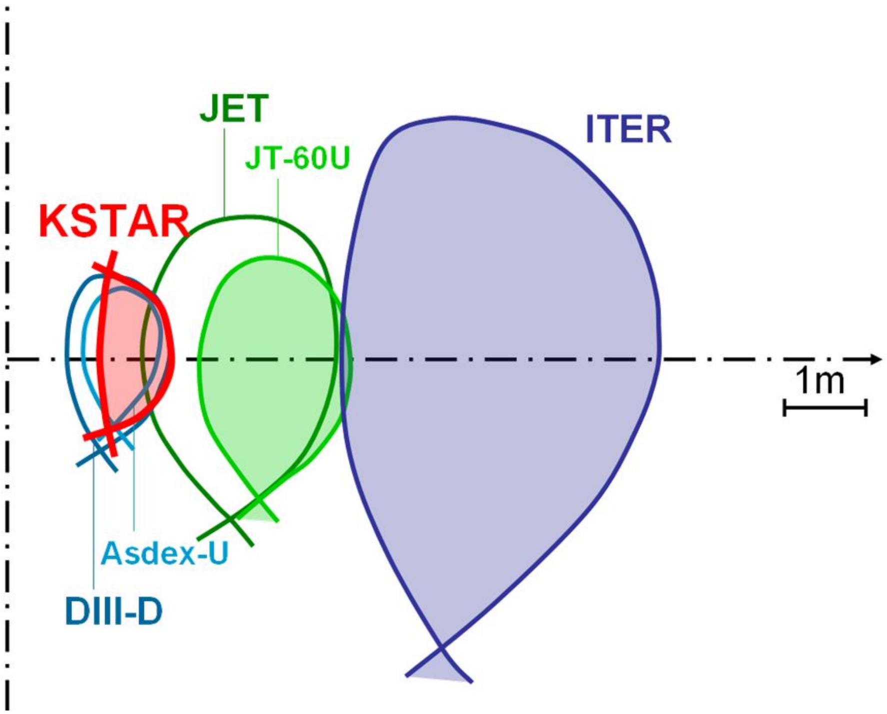
Tokamak: the most intensively studied magnetic confinement device

KSTAR



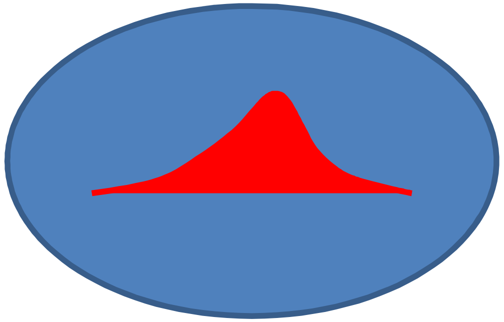
ITER





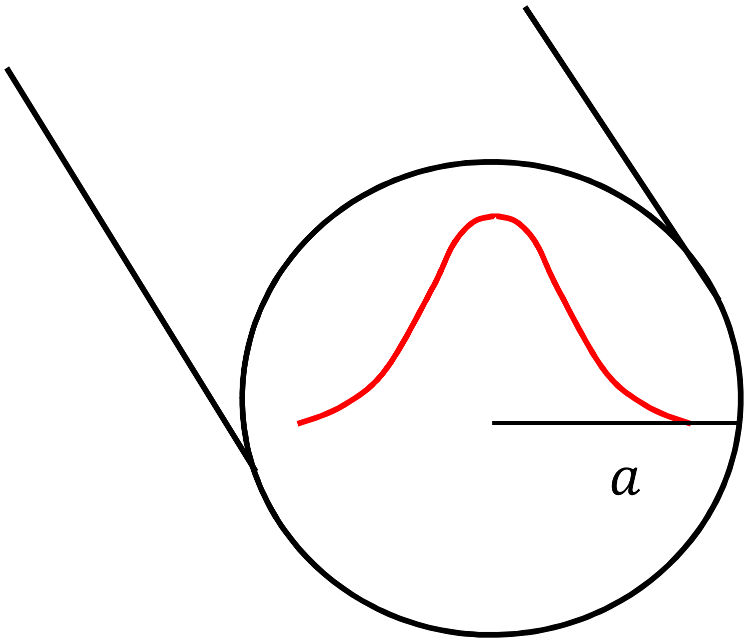
PARAMETERS	ITER	KSTAR
Major radius	6.2m	1.8m
Minor radius	2.0m	0.5m
Plasma volume	830m^3	17.8m^3
Plasma current	15MA	2.0MA
Toroidal field	5.3T	3.5T
Plasma fuel	H, D-T	H, D-D
Superconductor	Nb$_3$Sn, NbTi	Nb$_3$Sn, NbTi

Primer on Turbulence in Tokamaks I



- Strongly magnetized
  - Quasi 2D cells
  - Localized by $\vec{k} \cdot \vec{B} = 0$ (resonance)
- $\vec{V}_{\perp} = +\frac{c}{B}\,\vec{E} \times \hat{z}$
- ∇T_e, ∇T_i, ∇n driven
- Akin to thermal Rossby wave, with: g → magnetic curvature
- Resembles to wave turbulence, not high Re Navier-Stokes turbulence
- Re ill defined, $K \leq 1$

Primer on Turbulence in Tokamaks II



- $\nabla T, \nabla n$, etc. driver
- Quasi-2D, elongated cells aligned with B_0
- Characteristic scale ~ few ρ_i
- Characteristic velocity $v_d \sim \rho_* c_s$

2 scales:

$\rho \equiv$ gyro-radius

$a \equiv$ cross-section

$\rho_* \equiv \rho/a$ ➔ key ratio

- Transport scaling: $D \sim \rho v_d \sim \rho_* D_B \sim D_{GB}$
- i.e. Bigger is better! ➔ sets profile scale via heat balance
- Reality: $D \sim \rho_*^{\alpha} D_B$, $\alpha < 1$ ➔ why??

Transport: Local or Non-local?

- 40 years of fusion plasma modeling
  - local, diffusive transport

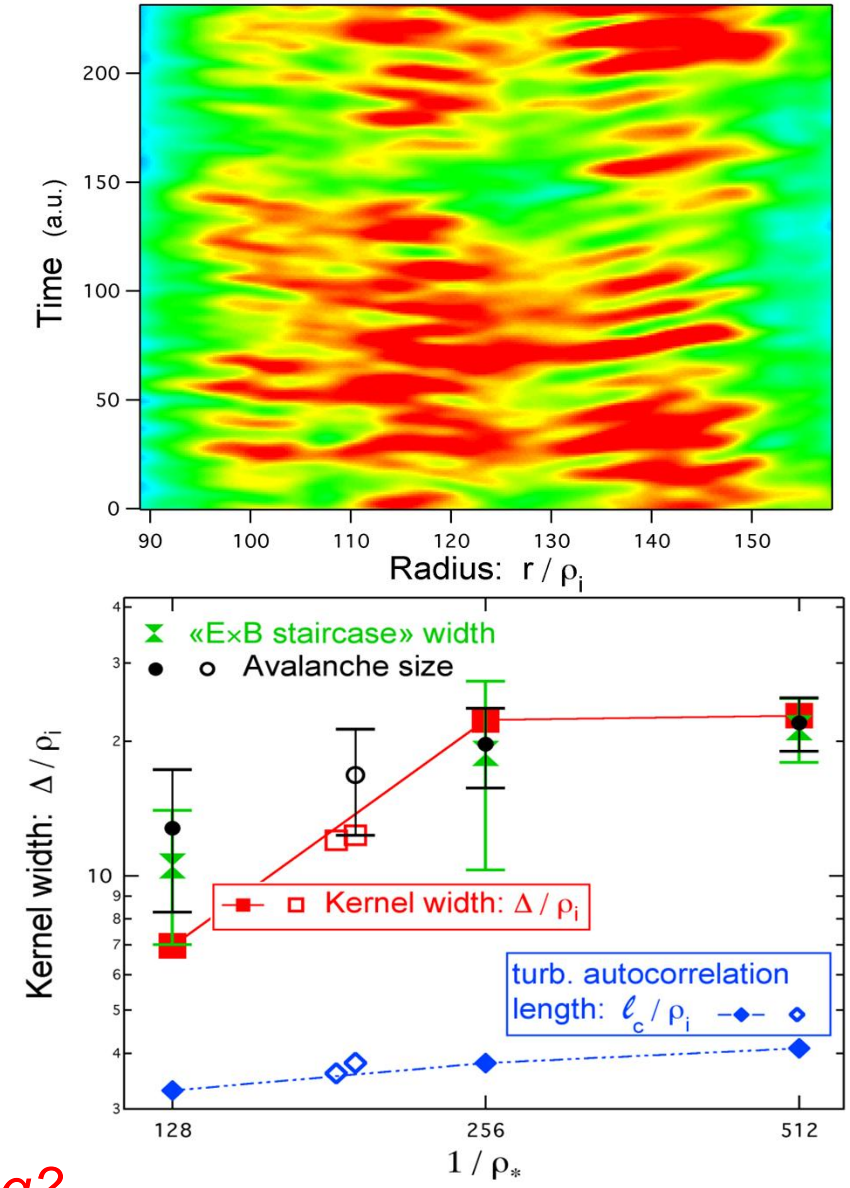
$$Q = -n\chi(r)\nabla T$$

- 1995 → increasing evidence for:
  - transport by avalanches as in sand pile/SOCs
  - turbulence propagation and invasion fronts
  - non-locality of transport

$$Q = -\int \kappa(r,r')\nabla T(r')dr'$$

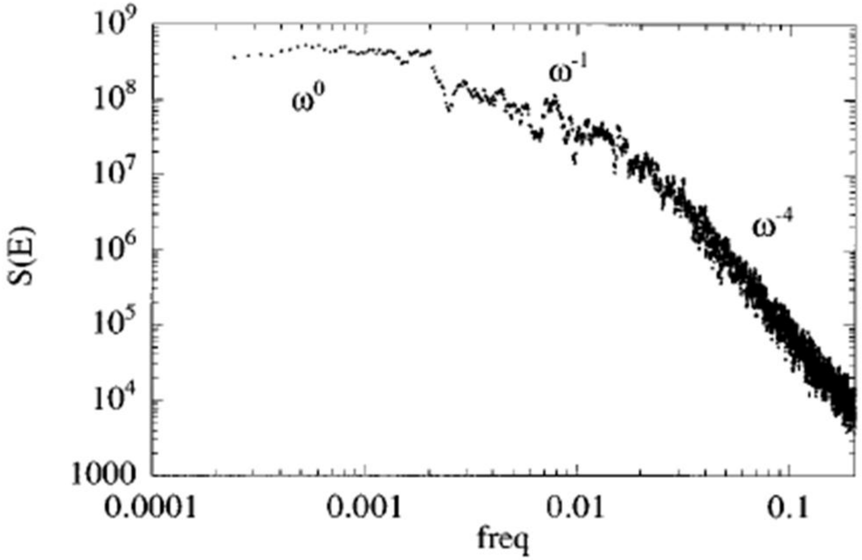
- Physics:
  - Levy flights, SOC, turbulence fronts…
- Fusion:
  - gyro-Bohm breaking
    (ITER: significant ρ_* extension)

→ *fundamentals of turbulent transport modeling?*

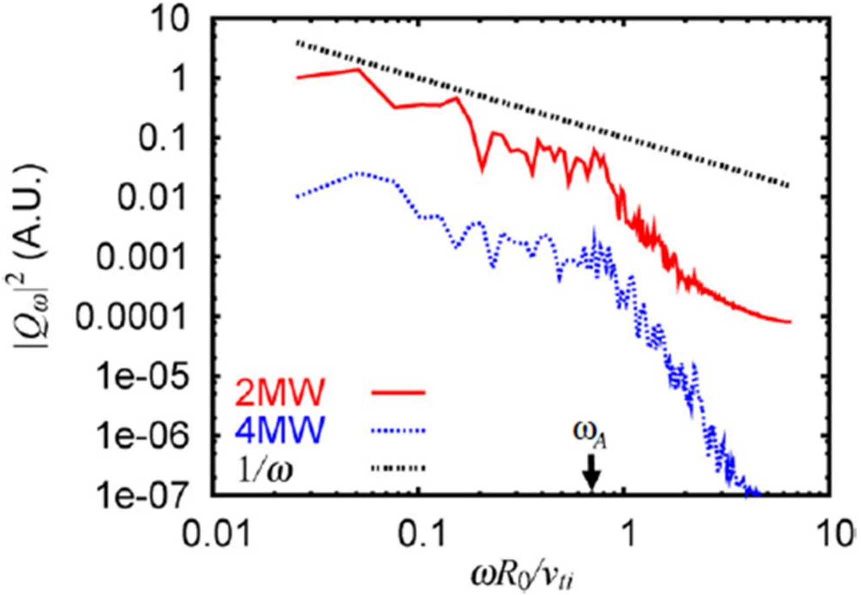


Guilhem Dif-Pradalier et al. PRL 2009

- ‘Avalanches’ form! – flux drive + geometrical ‘pinning’



Newman PoP96 (sandpile)
(Autopower frequency spectrum of ‘flip’)



GK simulation also exhibits avalanching
(Heat Flux Spectrum) (Idomura NF09)

- Avalanching is a likely cause of ‘gyro-Bohm breaking’

➔ localized cells self-organize to form transient, extended transport events

- Akin domino toppling:
- Pattern competition

with shear flows!

Toppling front can
penetrate beyond region
of local stability

- Cells “pinned” by magnetic geometry
- Remarkable Similarity:

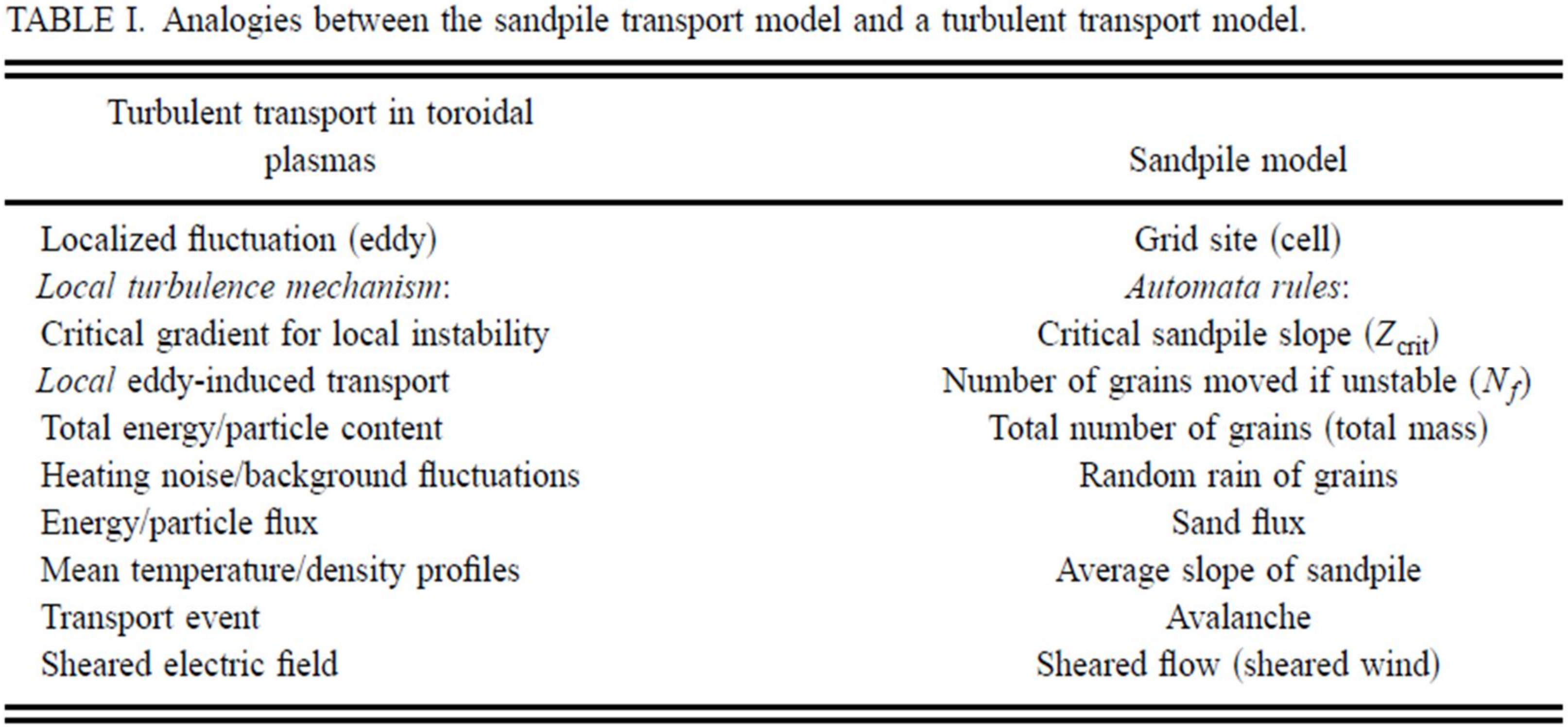
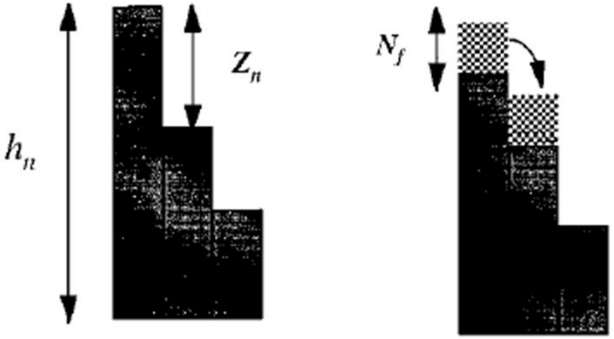


TABLE I. Analogies between the sandpile transport model and a turbulent transport model.

Turbulent transport in toroidal plasmas	Sandpile model
Localized fluctuation (eddy)	Grid site (cell)
Local turbulence mechanism:	*Automata rules*:
Critical gradient for local instability	Critical sandpile slope (Z_{crit})
Local eddy-induced transport	Number of grains moved if unstable (N_f)
Total energy/particle content	Total number of grains (total mass)
Heating noise/background fluctuations	Random rain of grains
Energy/particle flux	Sand flux
Mean temperature/density profiles	Average slope of sandpile
Transport event	Avalanche
Sheared electric field	Sheared flow (sheared wind)

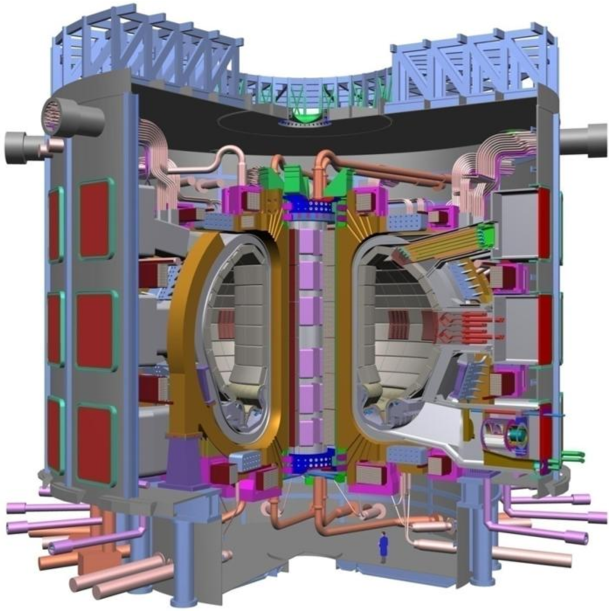
Automaton toppling
↔ Cell/eddy overturning

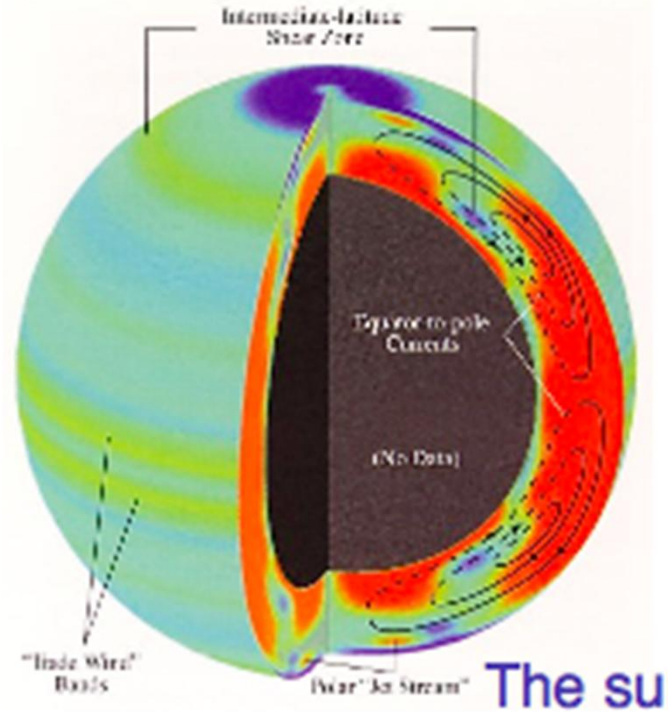


A cartoon representation of the simple cellular automata rules used to model the sandpile.

Preamble I

- Zonal Flows Ubiquitous for:
  - ~ 2D fluids / plasmas $R_0 < 1$
    Rotation $\vec{\Omega}$, Magnetization $\vec{B}_0$, Stratification
  - Ex: MFE devices, giant planets, stars…





The sun

Preamble II

- What is a Zonal Flow?
  - $n = 0$ potential mode; $m = 0$ (ZFZF), with possible sideband (GAM)
  - toroidally, poloidally symmetric *ExB* shear flow
- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport ($n = 0$)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

Zonal Flows I

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
    → Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation

- G.C. ambipolarity breaking → polarization charge flux → Reynolds force
  - Polarization charge → $-\boxed{\rho^2}\nabla^2\phi = \boxed{n_{i,GC}(\phi) - n_e(\phi)}$

    polarization length scale *ion GC* *electron density*

  - so $\Gamma_{i,GC} \neq \Gamma_e$ → $\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle \neq 0$ ↔ 'PV transport'

    polarization flux → What sets cross-phase?

  - If 1 direction of symmetry (or near symmetry):

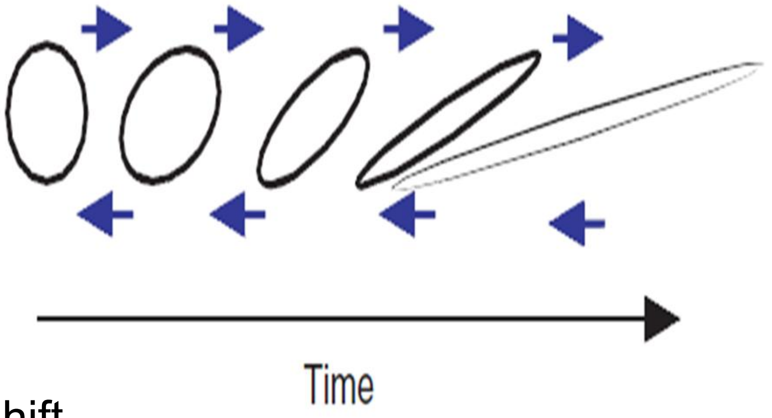
$$-\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle = -\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$$

(Taylor, 1915)

$-\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$ → Reynolds force → Flow

Shearing I

- Coherent shearing: (Kelvin, G.I. Taylor, Dupree'66, BDT'90)
  - radial scattering + $\langle V_E \rangle'$ → hybrid decorrelation
  - $k_r^2 D_\perp \rightarrow (k_\theta^2 \langle V_E \rangle'^2 D_\perp / 3)^{1/3} = 1/\tau_c$
  - shaping, flux compression: Hahm, Burrell '94



- Other shearing effects (linear):

Response shift
and dispersion

  - spatial resonance dispersion: $\omega - k_{\|} v_{\|} \Rightarrow \omega - k_{\|} v_{\|} - k_\theta \langle V_E \rangle' (r - r_0)$
  - differential response rotation → especially for kinetic curvature effects

→ N.B. Caveat: Modes can adjust to weaken effect of external shear

(Carreras, et. al. '92; Scott '92)

Shearing II

- Zonal Shears: Wave kinetics (Zakharov et. al.; P.D. et. al. '98, et. seq.)

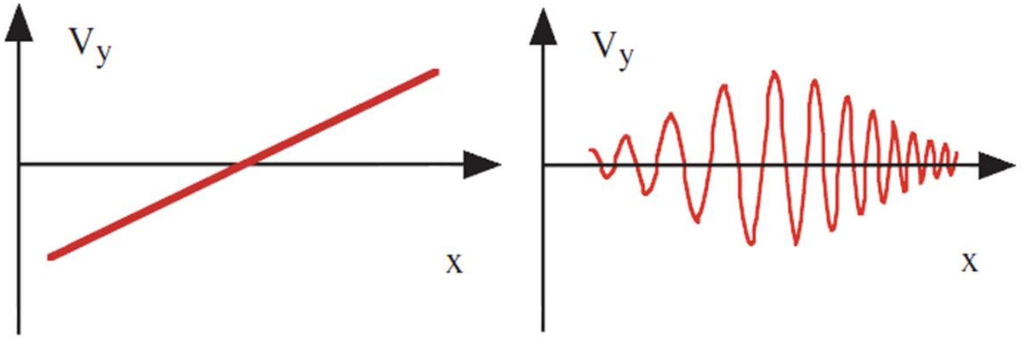
  Coherent interaction approach (L. Chen et. al.)

- $dk_r / dt = -\partial(\omega + k_\theta V_E)/\partial r$; $V_E = \langle V_E \rangle + \widetilde{V}_E$

  Mean shearing $: k_r = k_r^{(0)} - k_\theta V_E' \tau$

  Zonal Random shearing $: \langle \delta k_r^2 \rangle = D_k \tau$

  $$D_k = \sum_q k_\theta^2 \left| \widetilde{V}'_{E,q} \right|^2 \tau_{k,q}$$



- Wave ray chaos (not shear RPA) underlies $D_k \rightarrow$ induced diffusion
- Induces wave packet dispersion
- Applicable to ZFs and GAMs

- Mean Field Wave Kinetics

$$\frac{\partial N}{\partial t} + (\vec{V}_{gr} + \vec{V}) \cdot \nabla N - \frac{\partial}{\partial r}(\omega + k_\theta V_E) \cdot \frac{\partial N}{\partial \vec{k}} = \gamma_{\vec{k}} N - C\{N\}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t}\langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r} \langle N \rangle = \gamma_{\vec{k}} \langle N \rangle - \langle C\{N\} \rangle}$$

Zonal shearing

Shearing III

- Energetics: Books Balance for Reynolds Stress-Driven Flows!

- Fluctuation Energy Evolution – Z.F. shearing

$$\int d\vec{k}\,\omega\left(\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r}D_k\frac{\partial}{\partial k_r}\langle N\rangle\right) \Rightarrow \frac{\partial}{\partial t}\langle \varepsilon\rangle = -\int d\vec{k}\,V_{gr}(\vec{k})D_{\vec{k}}\frac{\partial}{\partial k_r}\langle N\rangle \qquad V_{gr} = \frac{-2k_r k_\theta V_* \rho_s^2}{\left(1+k_\perp^2\rho_s^2\right)^2}$$

Point: For $d\langle\Omega\rangle / dk_r < 0$, Z.F. shearing damps wave energy

- Fate of the Energy: Reynolds work on Zonal Flow

Modulational Instability

$$\partial_t \delta V_\theta + \partial\left(\delta\left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle\right) / \partial r = -\gamma \delta V_\theta$$

$$\delta\left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle \sim \frac{k_r k_\theta \delta\Omega}{(1+k_\perp^2\rho_s^2)^2}$$

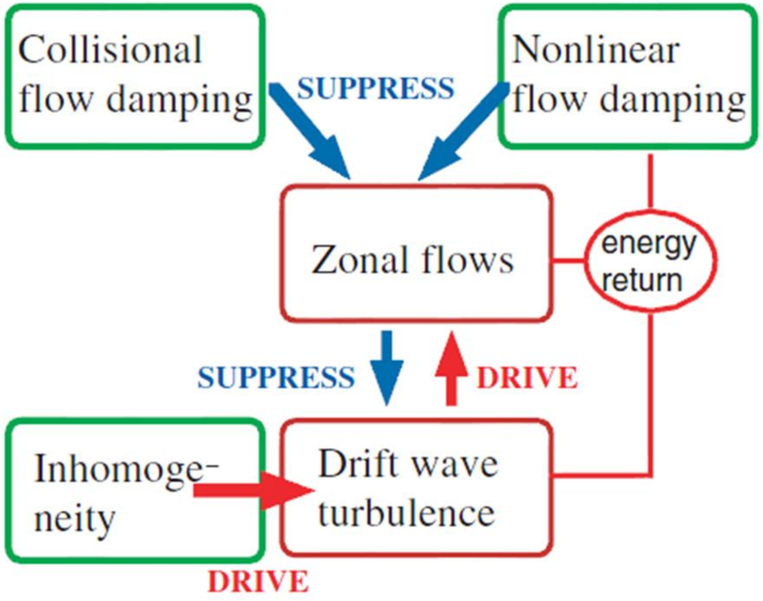
N.B.: Wave decorrelation essential:
Equivalent to PV transport
(c.f. Gurcan et. al. 2010)

- Bottom Line:
  - Z.F. growth due to shearing of waves
  - “Reynolds work” and “flow shearing” as relabeling → books balance
  - Z.F. damping emerges as critical; MNR ‘97

Feedback Loops I

- Closing the loop of shearing and Reynolds work

- Spectral ‘Predator-Prey’ equations


Collisional flow damping

SUPPRESS

Nonlinear flow damping

Zonal flows

energy return

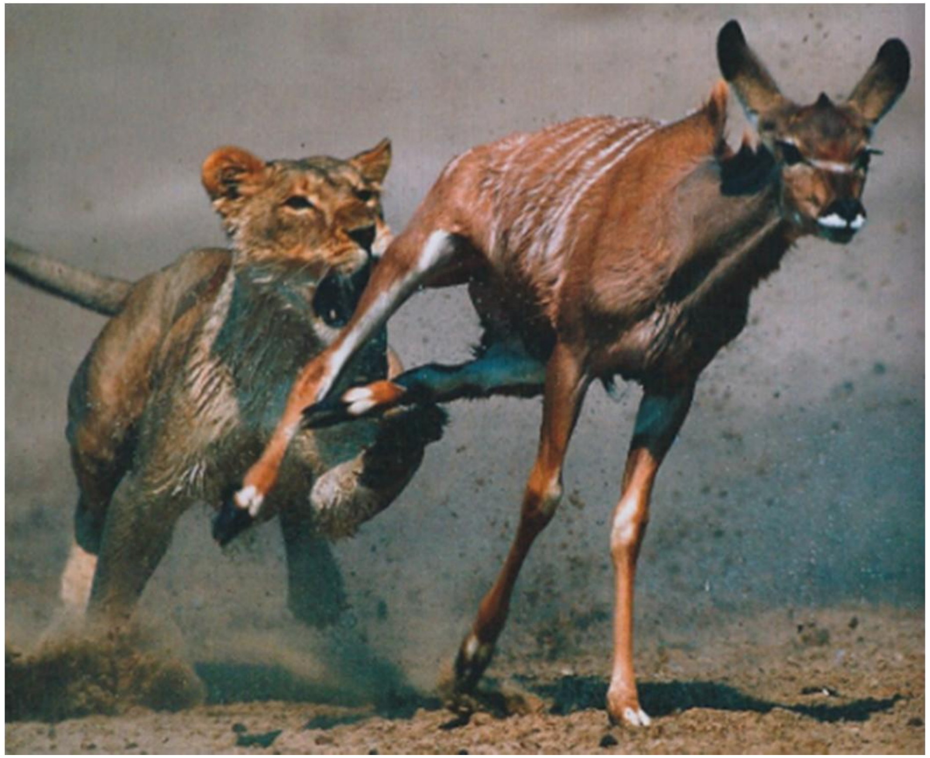
SUPPRESS

DRIVE

Inhomogeneity

Drift wave turbulence

DRIVE



Prey → Drift waves, <*N*>

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k\langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

Feedback Loops II

- Recovering the ‘dual cascade’:
  - Prey → <N> ~ <Ω> ⇒ induced diffusion to high k_r $\left\{ \begin{array}{l} \Rightarrow \text{Analogous} \rightarrow \text{forward potential} \\ \text{enstrophy cascade; PV transport} \end{array} \right.$
  - Predator → $|\phi_q|^2 \sim \langle V_{E,\theta}^2 \rangle$ $\left\{ \begin{array}{l} \Rightarrow \text{growth of } n=0,\ m=0 \text{ Z.F. by turbulent Reynolds work} \\ \Rightarrow \text{Analogous} \rightarrow \text{inverse energy cascade} \end{array} \right.$

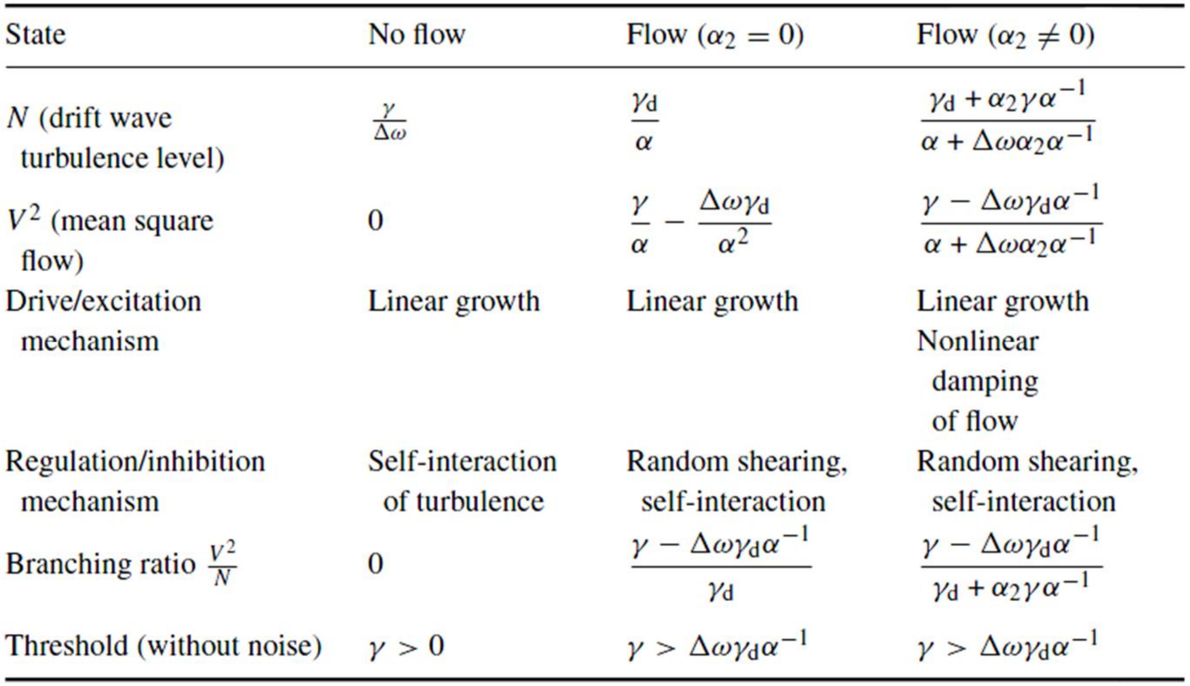
- Mean Field Predator-Prey Model

(P.D. et. al. ’94, DI²H ‘05)

$$\frac{\partial}{\partial t} N = \gamma N - \alpha V^2 N - \Delta\omega N^2$$

$$\frac{\partial}{\partial t} V^2 = \alpha N V^2 - \gamma_d V^2 - \gamma_{NL}(V^2) V^2$$

System Status



State	No flow	Flow ($\alpha_2 = 0$)	Flow ($\alpha_2 \neq 0$)
N (drift wave turbulence level)	$\frac{\gamma}{\Delta\omega}$	$\frac{\gamma_d}{\alpha}$	$\frac{\gamma_d + \alpha_2 \gamma \alpha^{-1}}{\alpha + \Delta\omega \alpha_2 \alpha^{-1}}$
V^2 (mean square flow)	0	$\frac{\gamma}{\alpha} - \frac{\Delta\omega\gamma_d}{\alpha^2}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
Drive/excitation mechanism	Linear growth	Linear growth	Linear growth Nonlinear damping of flow
Regulation/inhibition mechanism	Self-interaction of turbulence	Random shearing, self-interaction	Random shearing, self-interaction
Branching ratio $\frac{V^2}{N}$	0	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d + \alpha_2\gamma\alpha^{-1}}$
Threshold (without noise)	$\gamma > 0$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$

Observations

- Fundamental concept of zonal flow formation is secondary mode in gas of drift waves, i.e. modulational instability
  - wave kinetics
  - envelope expansion
  - …
- N.B. No clear scale separation, inverse cascade, Rhines mechanism …
- Interest is driven by (favorable) impact of flows on confinement
- This drives a concern with feedback and the picture of co-existing, competing populations, etc.

A Central Question: Secondary Pattern Selection

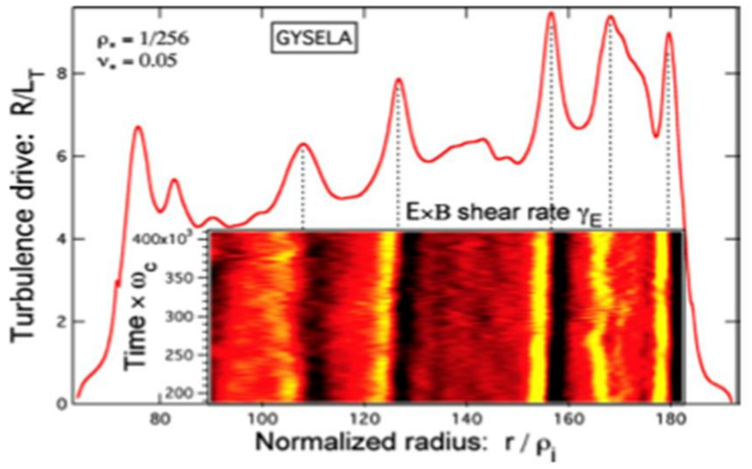
- Two secondary structures suggested
  - Zonal flow → quasi-coherent, regulates transport via shearing
  - Avalanche → stochastic, induces extended transport events
- Nature of co-existence?

Staircases and Traffic Jams

Single Barrier → Lattice of Shear Layers

→ Jam Patterns

Highlights



Observation of ExB staircases

→ Failure of conventional theory

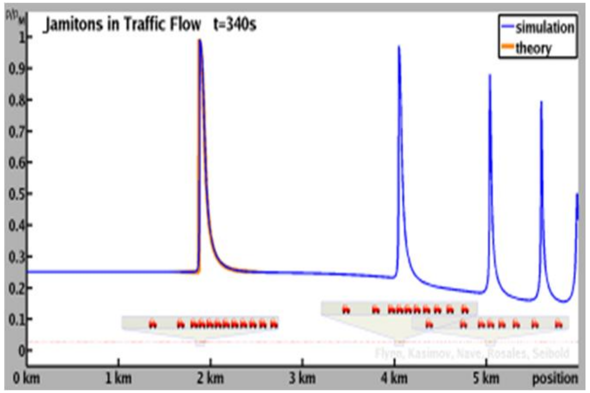
(emergence of particular scale???)

Model extension from Burgers to telegraph

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

$$\Rightarrow \boxed{\tau \partial_t^2 \delta T} + \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

finite response time → like drivers' response time in traffic



Analysis of telegraph eqn. predicts heat flux jam

- scale of jam comparable to staircase step

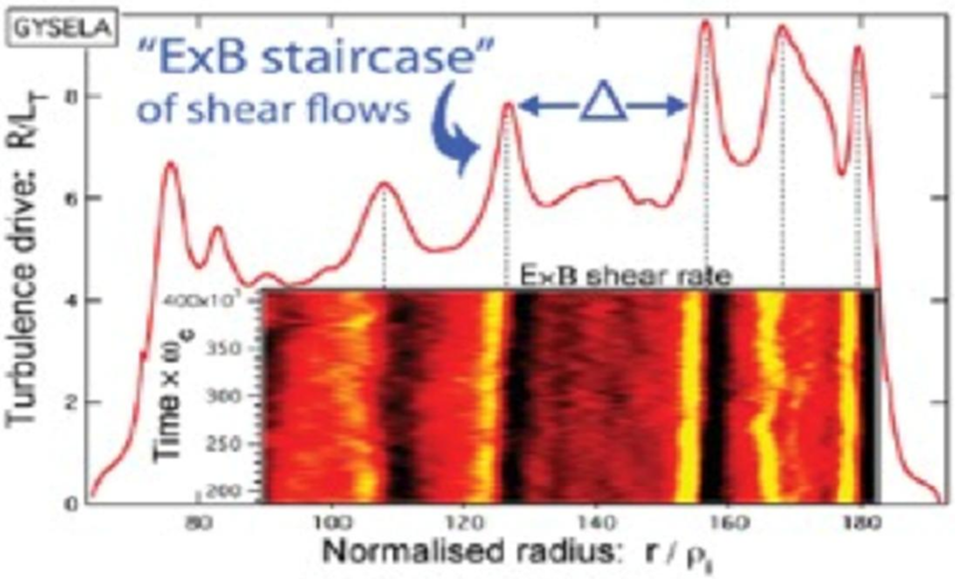
Motivation: ExB staircase formation (1)

- ExB flows often observed to self-organize in magnetized plasmas

  eg.) mean sheared flows, zonal flows, ...

- \`ExB staircase’ is observed to form

(G. Dif-Pradalier, P.D. et al. Phys. Rev. E. ’10)



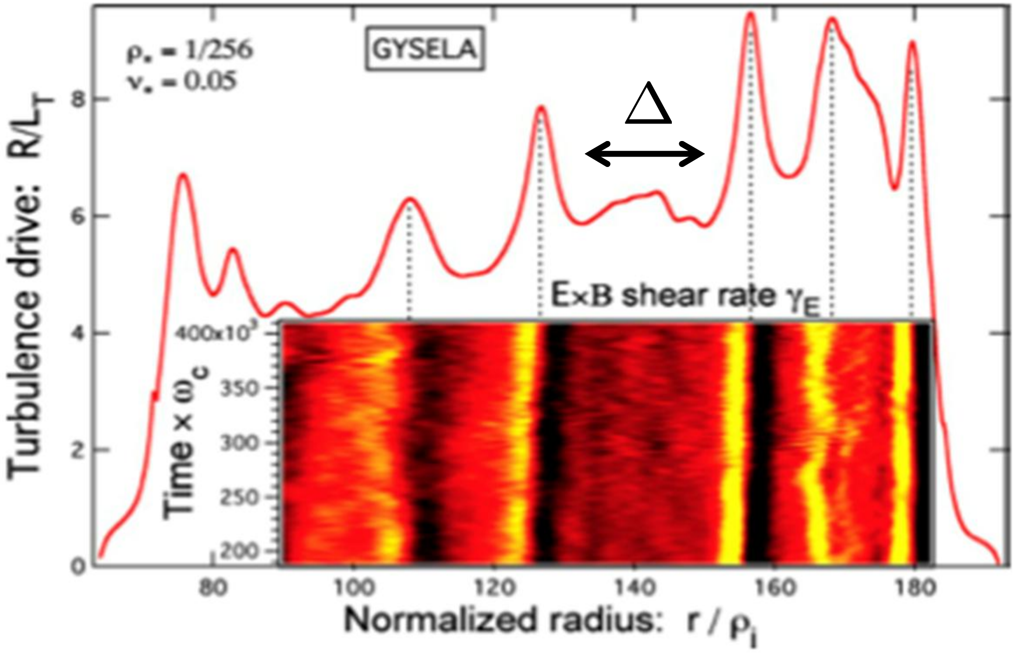
[from Dunkerton et al. 2008]

- flux driven, full f simulation
- Quasi-regular pattern of shear layers and profile corrugations
- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

  → ExB staircases
- so-named after the analogy to PV staircases and atmospheric jets
- Step spacing → avalanche outer-scale

ExB Staircase (2)

- Important feature: co-existence of shear flows and avalanches



- Seem mutually exclusive ?!?

→ strong ExB shear prohibits transport

→ avalanches smooth out corrugations

- Can co-exist by separating regions into:

1. avalanches of the size $\Delta \gg \Delta_c$

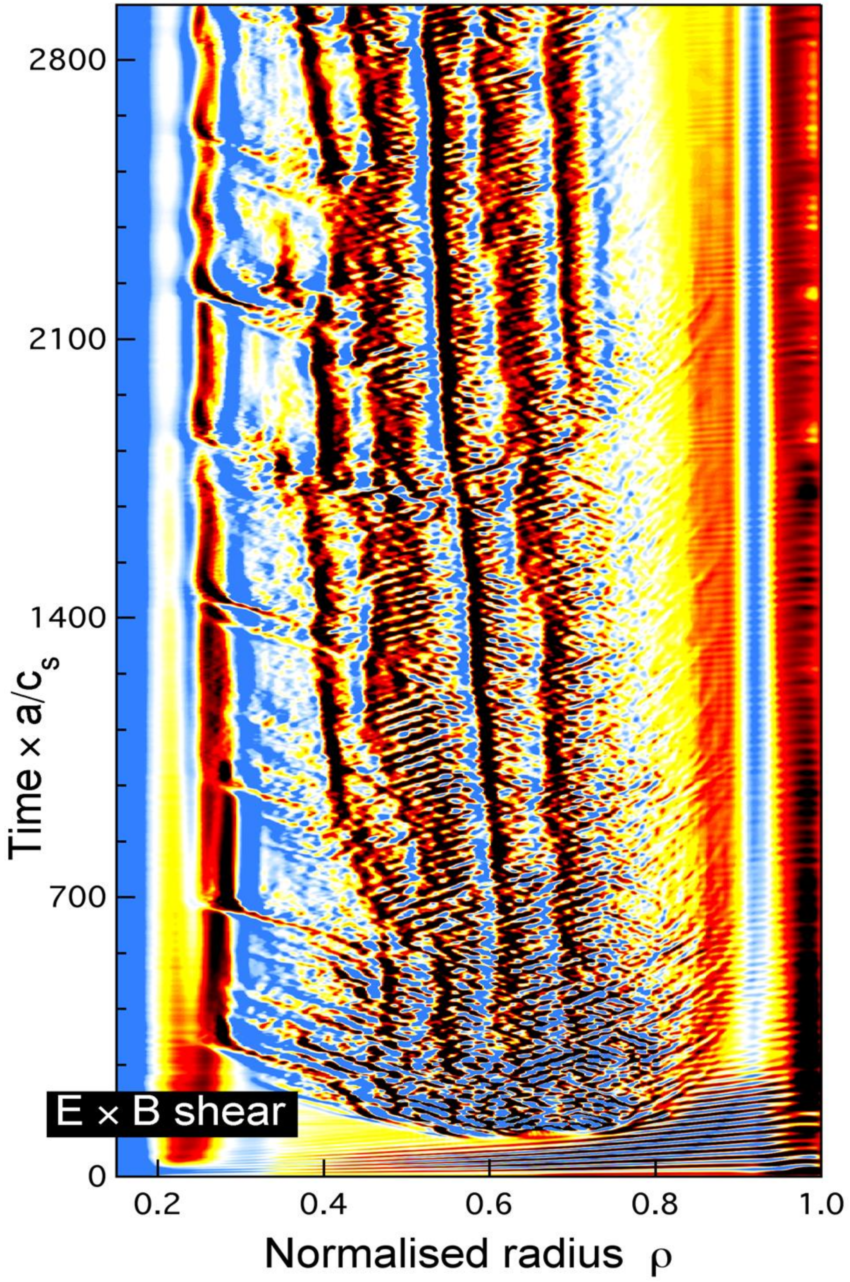
2. localized strong corrugations + jets

- How understand the formation of ExB staircase???

- What is process of self-organization linking avalanche scale to ExB step scale?

i.e. how explain the emergence of the step scale ???

Staircases build up from the edge

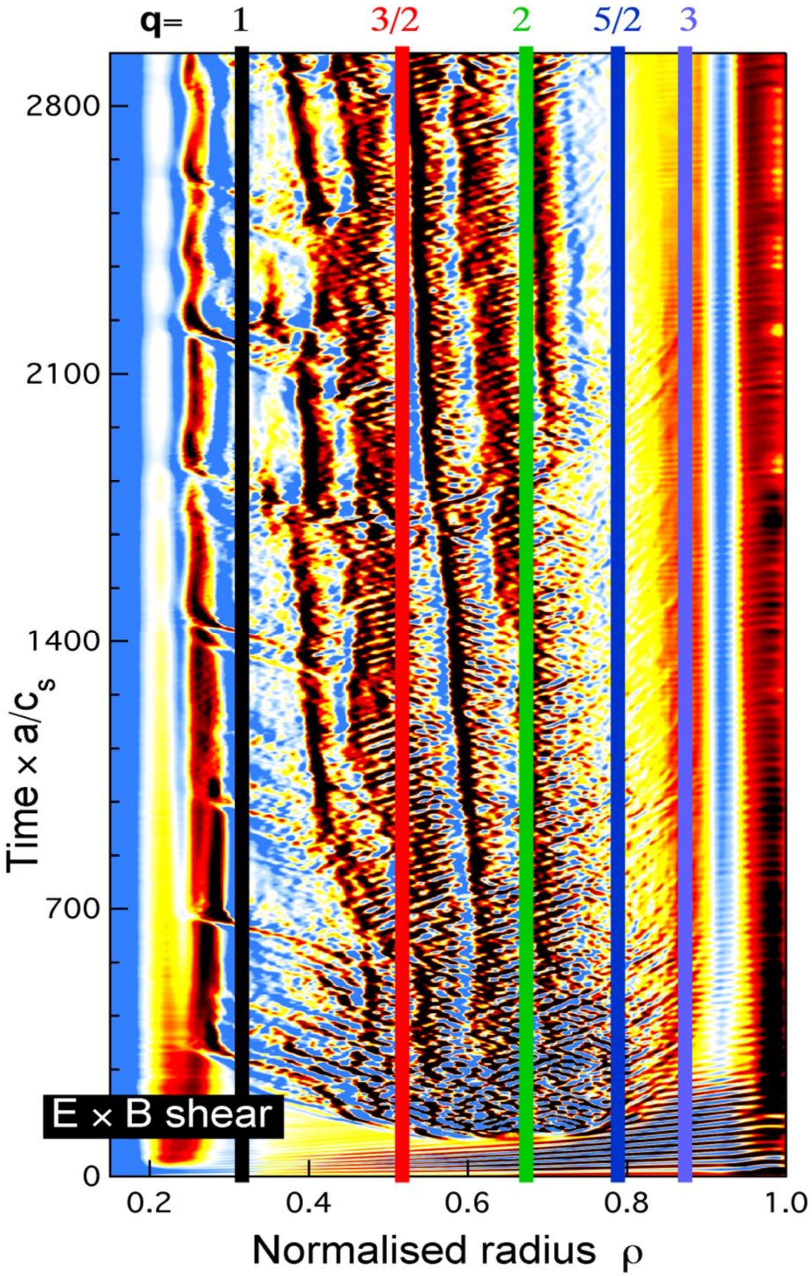


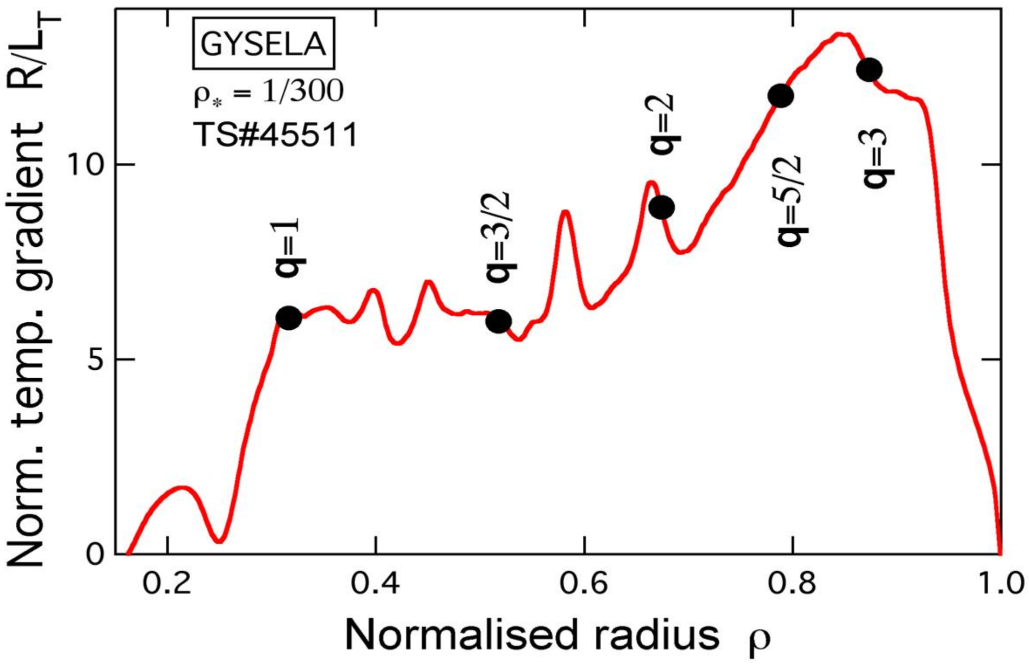
→ staircases may not be related to zonal flow eigenfunctions

→ How describe generation mechanism??

(GYSELA simulation)

Corrugation points and rational surfaces – no relation!





Step location not tied to magnetic geometry structure in a simple way

Towards a model

- How do we understand quasi-regular pattern of ExB staircase, generated from stochastic heat avalanche???

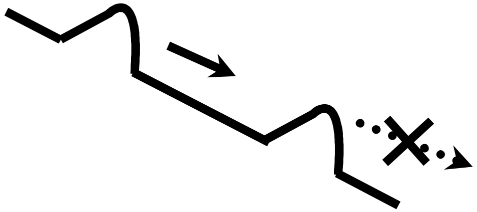
- An idea: jam of heat avalanche

corrugated profile ⟷ ExB staircase

→ corrugation of profile occurs by 'jam' of heat avalanche flux

* → time delay between $Q[\delta T]$ and δT is crucial element

like drivers' response time in traffic



→ accumulation of heat increment
→ stationary corrugated profile

- How do we actually model heat avalanche 'jam' ??? → origin in dynamics?

Traffic jam dynamics: ‘jamiton’

- A model for Traffic jam dynamics → Whitham

$$\rho_t + (\rho v)_x = 0$$

$$v_t + v v_x = -\frac{1}{\tau}\left\{v - V(\rho) + \frac{\nu}{\rho}\rho_x\right\}$$

→ Instability occurs when $\tau > \nu/(\rho_0^2 V_0'^2)$

$$D_{eff} = \nu - \tau\rho_0^2 V_0'^2 < 0$$ → clustering instability

→ Indicative of jam formation

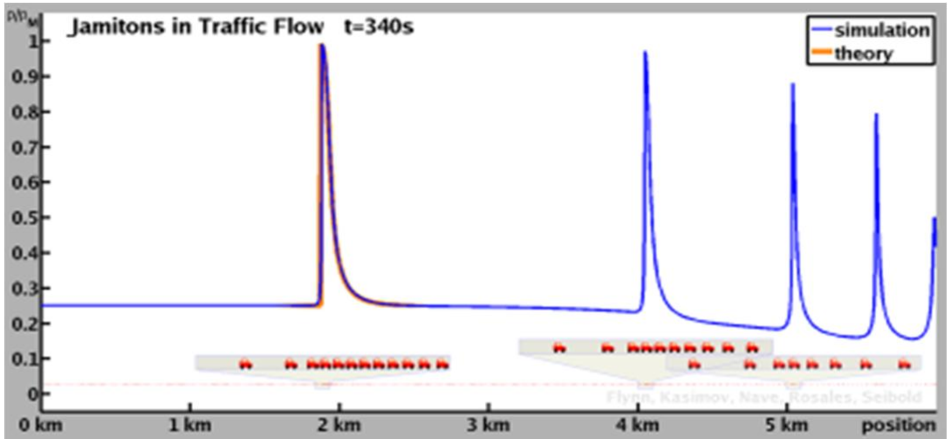
ρ → car density

v → traffic flow velocity

$V(\rho) - \frac{\nu}{\rho}\rho_x$ → an equilibrium traffic flow

τ → driver’s response time

- Simulation of traffic jam formation



http://math.mit.edu/projects/traffic/

→ Jamitons (Flynn, et.al., ’08)

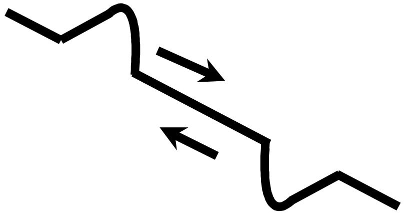
n.b. I.V.P. → decay study

Heat avalanche dynamics model (`the usual’)

Hwa+Kardar ’ 92, P.D. + Hahm ’ 95, Carreras, et al. ’ 96, ... GK simulation, ... Dif-Pradalier ’ 10

- δT :deviation from marginal profile → conserved order parameter
- Heat Balance Eq.: $\partial_t \delta T + \partial_x Q[\delta T] = 0$ → up to source and noise
- Heat Flux $Q[\delta T]$? → utilize symmetry argument, ala’ Ginzburg-Landau

  - Usual: → joint reflectional symmetry (Hwa+Kardar’92, Diamond+Hahm ’95)



$$\delta T \leftrightarrow -\delta T$$
$$x \leftrightarrow -x$$

$$\begin{aligned} Q &= Q_0(\delta T) \\ &= \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T \end{aligned}$$

→ hyperdiffusion

lowest order → Burgers equation

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

An extension of the heat avalanche dynamics

- An extension: a finite time of relaxation of Q toward SOC flux state

$$\partial_t Q = -\frac{1}{\tau}(Q - Q_0(\delta T)) \qquad Q_0[\delta T] = \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

(Guyot-Krumhansl)

→ In principle $\tau(\delta T, Q_0)$ ⟷ large near criticality (∼ critical slowing down)

i.e. enforces time delay between δT and heat flux

- Dynamics of heat avalanche:

$$\underline{\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T} - \chi_4 \partial_x^4 \delta T - \underset{\downarrow}{\tau \partial_t^2 \delta T}$$

→ Burgers
(P.D. + T.S.H. ’95)

New: finite response time

→ Telegraph equation

n.b. model for heat evolution

diffusion → Burgers → Telegraph

Relaxation time: the idea

- What is 'τ ' physically? → Learn from traffic jam dynamics
- A useful analogy:

heat avalanche dynamics	traffic flow dynamics
temp. deviation from marginal profile	local car density
heat flux	traffic flow
mean SOC flux (ala joint relflection symmetry)	equilibrium, steady traffic flow
➡ heat flux relaxation time	driver's response time

- driver's response can induce traffic jam
- jam in avalanche → profile corrugation → staircase?!?
- Key: instantaneous flux vs. mean flux

Heat flux dynamics: when important?

- Heat flux evolution: $\partial_t Q = -\frac{1}{\tau_{mix}}(Q - Q_0)$ → time delay, when important?

Conventional Transport Analysis

$\tau_{mix} \ll$ time scale of interest

→ Heat flux relaxes to the mean value immediately

$$Q = Q_0$$

→ Profile evolves via the mean flux

$$\partial_t T + \partial_x Q_0 = 0$$

then

diff. $\partial_t T = \chi \partial_x^2 T$

Burgers $\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$

New approach for transport analysis

→ mixing time can be long, so

$\tau_{mix} \sim$ time scale of interest
mesoscale

→ Heat evo. and Profile evo. must be treated self-consistently

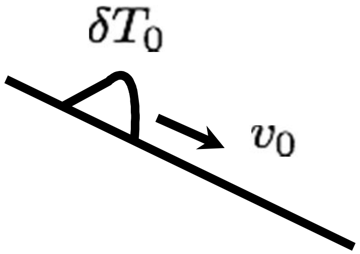
$$\begin{cases} \partial_t Q = -\frac{1}{\tau}(Q - Q_0) \\ \partial_t \delta T + \partial_x Q[\delta T] = 0 \end{cases}$$

then telegraph equation:

$$\boxed{\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \tau \partial_t^2 \delta T}$$

Analysis of heat avalanche dynamics via telegraph

- How do heat avalanches jam?



- Consider an initial avalanche,
  with amplitude δT_0,
  propagating at the speed $v_0 = \lambda \delta T_0$

→ turbulence model dependent

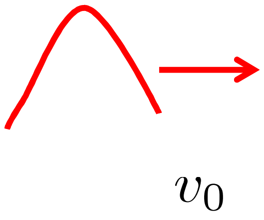
- Dynamics:

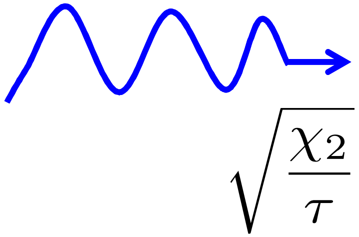
$$\partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} = \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T}$$

pulse

'Heat flux wave': $\sqrt{\frac{\chi_2}{\tau}}$
telegraph → wavy feature

two characteristic propagation speeds





→ In short response time (usual) heat flux wave propagates faster

→ In long response time, heat flux wave becomes slower and pulse starts overtaking.
What happens???

Analysis of heat avalanche jam dynamics

- In large tau limit, what happens? → Heat flux jams!!

- Recall plasma response time akin to driver's response time in traffic dynamics

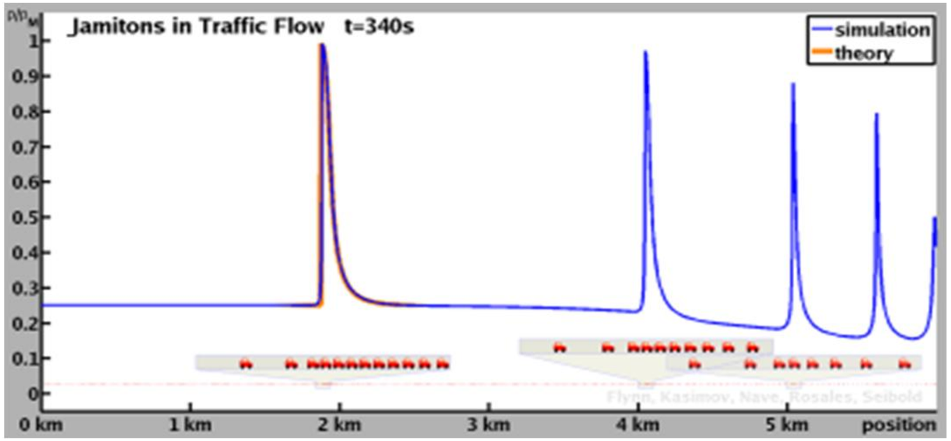
- negative heat conduction instability occurs (as in clustering instability in traffic jam dynamics)

$$\partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} = \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T}$$

$$\rightarrow \underline{(\chi_2 - v_0^2 \tau)} \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T}$$

<0 when overtaking

→ clustering instability



n.b. akin to negative viscosity instability of ZF in DW turbulence

instead ZF as secondary mode in the gas of primary DW

➔ Heat flux 'jamiton' as secondary mode in the gas of primary avalanches

Analysis of heat avalanche jam dynamics

- Growth rate of the jamiton instability

$$\gamma = -\frac{1}{2\tau} + \frac{1}{2\tau}\sqrt{\frac{r+1}{2} - 2\tau\chi_2 k^2\left(1+\frac{\chi_4 k^2}{\chi_2}\right)} \qquad r = \sqrt{\left\{4\tau\chi_2 k^2\left(1+\frac{\chi_4 k^2}{\chi_2}\right) - 1\right\}^2 + 16 v_0^2 k^2 \tau^2}$$

- Threshold for instability

$$\tau > \frac{\chi_2}{v_0^2}\left(1+\frac{\chi_4 k^2}{\chi_2}\right)$$

→ critical minimal delay time

n.b. $1/\tau = 1/\tau[\mathcal{E}]$

→ clustering instability strongest near criticality

- Scale for maximum growth

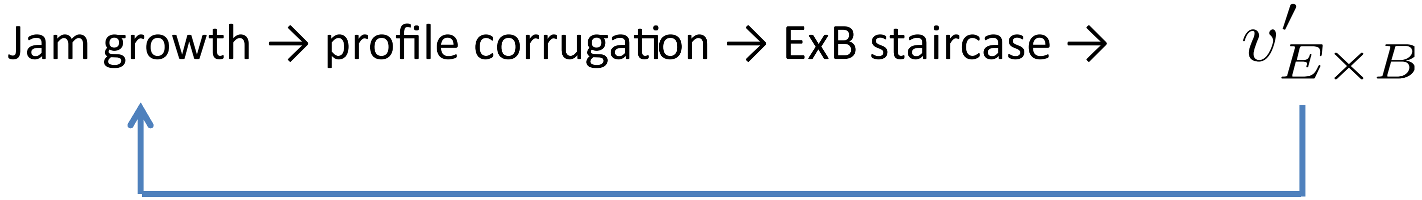
$$k^2 \cong \frac{\chi_2}{\chi_4}\sqrt{\frac{\chi_4 v_0^2}{4\chi_2^3}}$$

from $\frac{\partial \gamma}{\partial k^2} = 0 \Rightarrow 8\tau\frac{\chi_4^2}{\chi_2}k^6 + 4\tau\chi_4 k^4 + 2\frac{\chi_4}{\chi_2}k^2 + 1 - \frac{v_0^2\tau}{\chi_2} = 0$

→ staircase size, $\Delta_{stair}^2(\delta T)$, δT from saturation: consider shearing

Scaling of characteristic jam scale

- Saturation: Shearing strength to suppress clustering instability



Jam growth → profile corrugation → ExB staircase → $v'_{E\times B}$

→ estimate, only

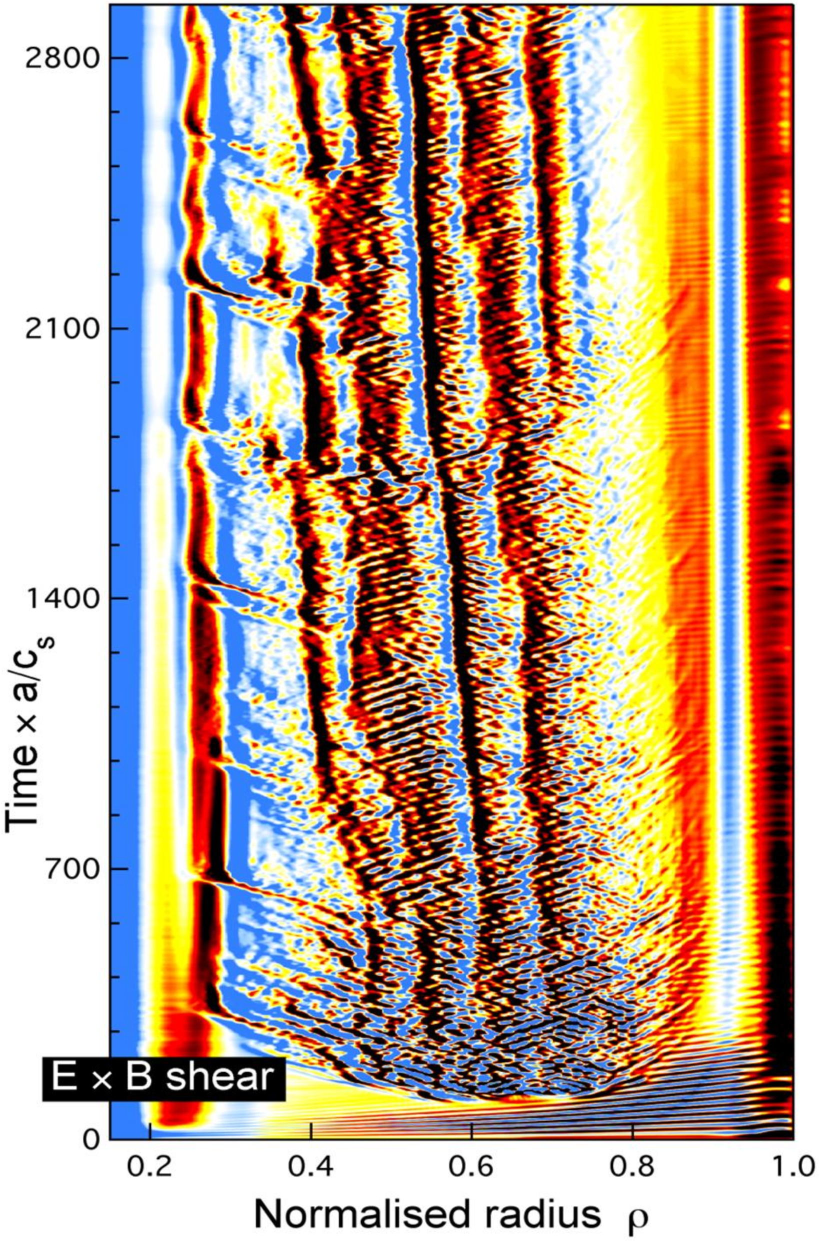
→ saturated amplitude: $\dfrac{\delta T}{T_i} \sim \dfrac{1}{v_{thi}\rho_i}\sqrt{\dfrac{\chi_4}{\tau}}$

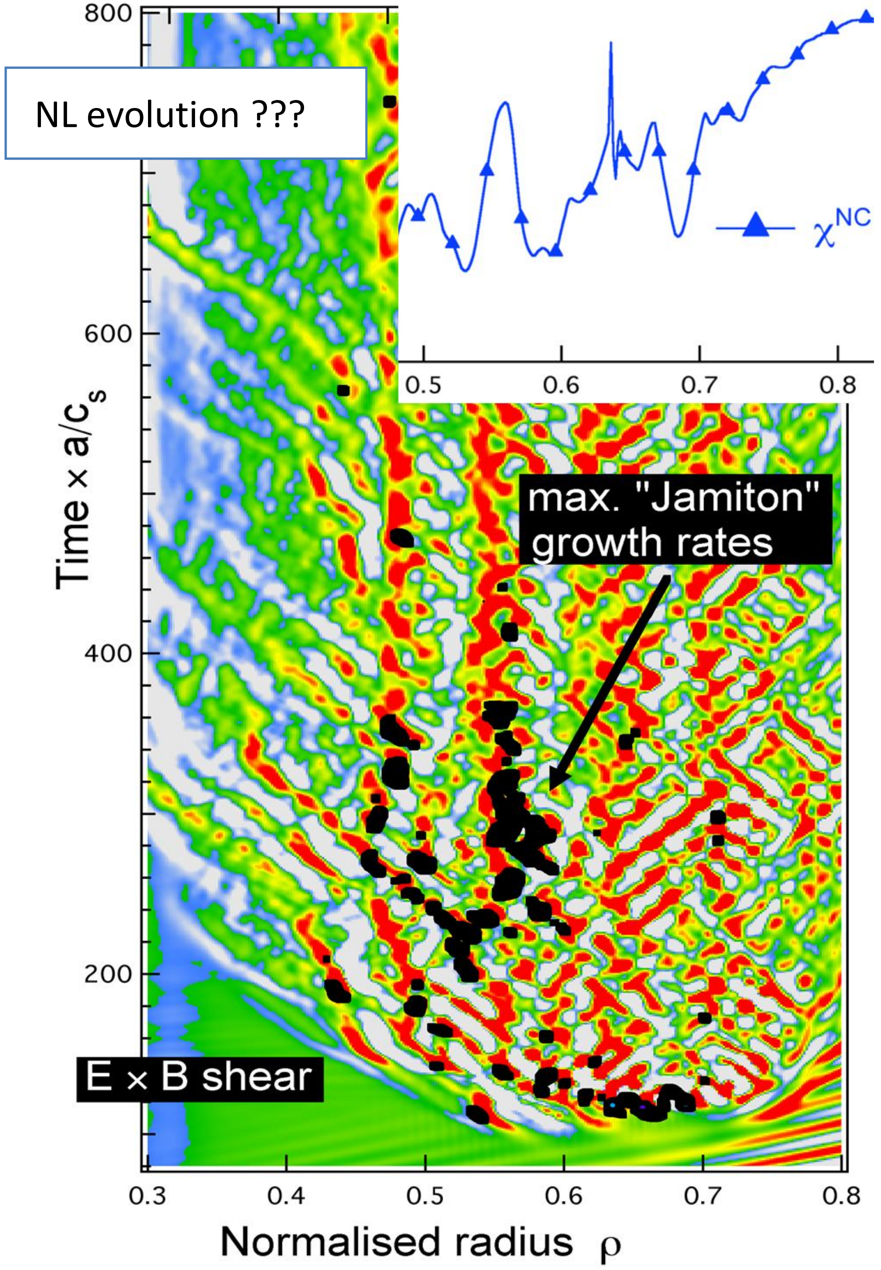
- Characteristic scale

$$\Delta^2 \sim k^{-2}(\delta T) \sim \frac{2v_{thi}}{\lambda T_i}\rho_i\sqrt{\chi_2\tau} \qquad \chi_2 \sim \chi_{neo}$$

- Geometric mean of ρ_i and $\sqrt{\chi_2\tau}$: ambient diffusion length in 1 relaxation time
- ‘standard’ parameters: $\Delta \sim 10\Delta_c$

Jam growth qualitatively consistent with staircase formation




NL evolution ???

outer radius:
large chi
→ smear out instability
or
→ heat flux waves propagate faster
→ harder to overtake, jam

good agreement in early stage

Dif-Pradalier ’13 caveat: based on model with compressional waves

$E \times B$ staircase visible on fluctuation correlations

Direct exp. characterisation difficult:

flows, profiles & gradients

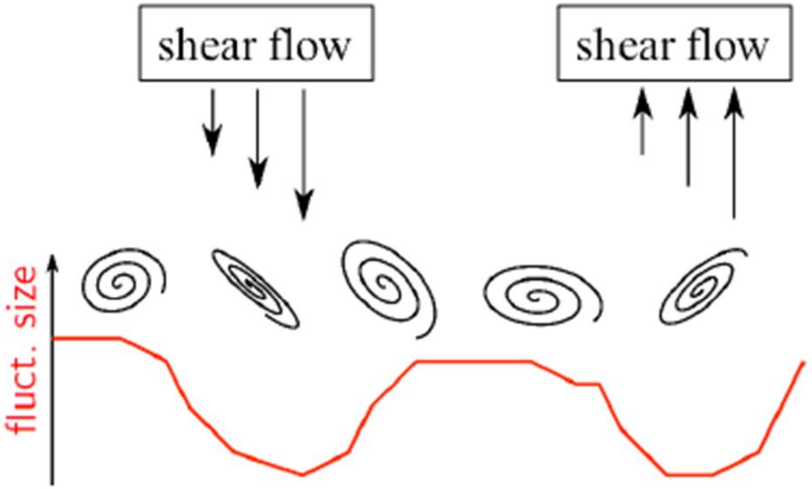
Shear layers in staircase:

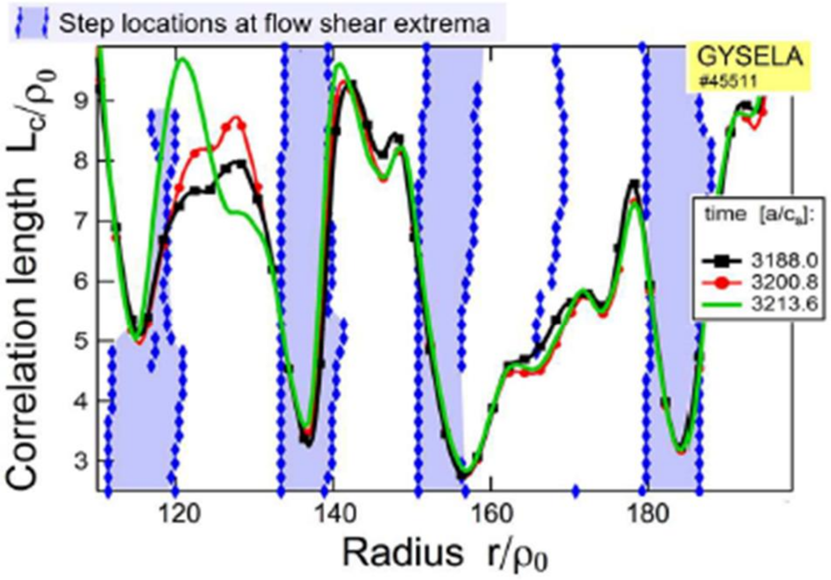
- eddies stretched, tilted, fragmented
- predict quasi-periodic decorrelation turbulent fluct.

$$\mathcal{C}_\phi(r,\theta,t,\delta r) = \frac{\langle \tilde{\phi}(r,\theta,t)\ \tilde{\phi}(r+\delta r,\theta,t)\rangle_\tau}{\left[\langle \tilde{\phi}(r,\theta,t)^2\rangle_\tau\ \langle \tilde{\phi}(r+\delta r,\theta,t)^2\rangle_\tau\right]^{1/2}}$$

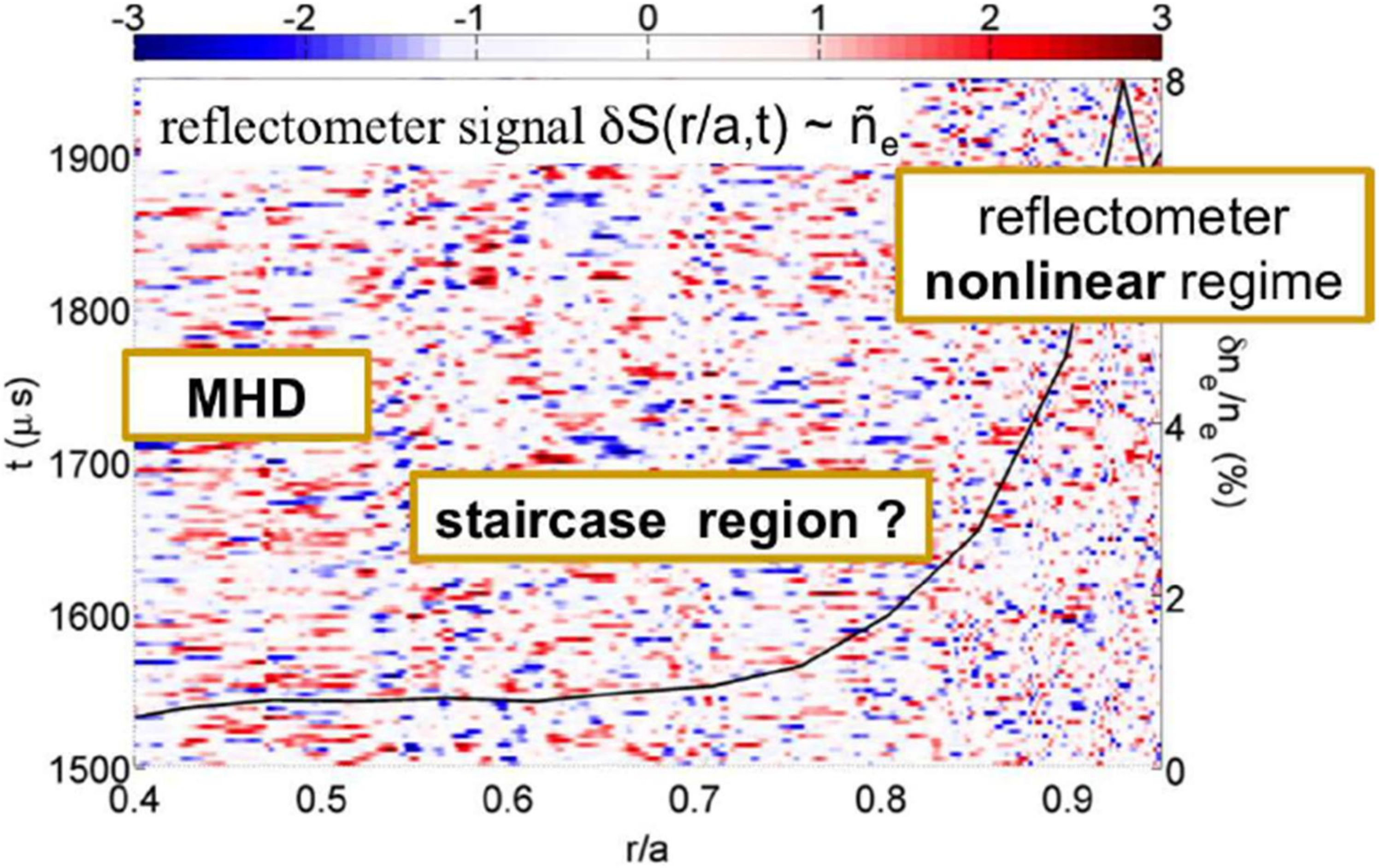
➥ $\mathcal{C}_\phi = 1/2$ when $\delta r = L_c$

➥ testable with fast-sweeping reflectometry





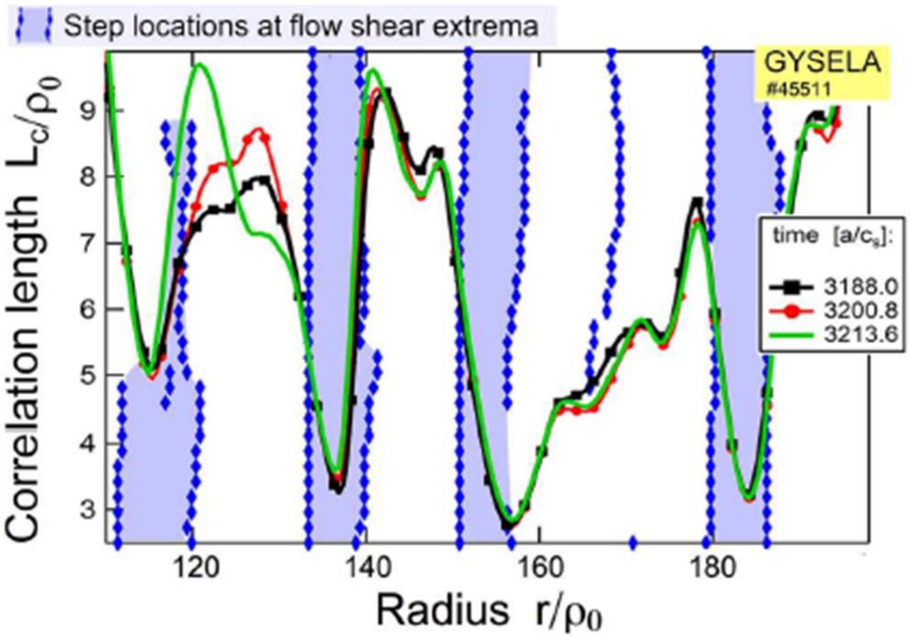
Moderate fluctuation level & MHD-free plasmas: optimal for staircase observation

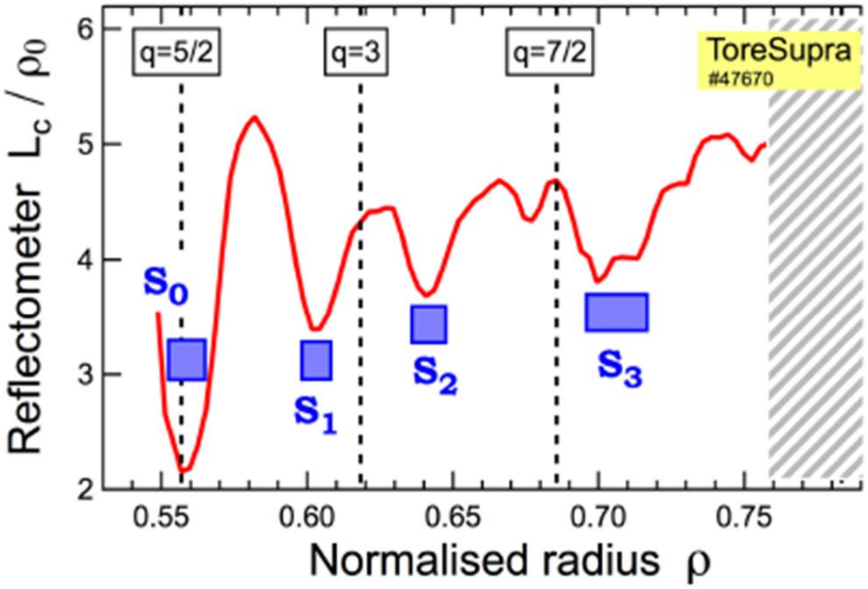


fast-sweeping reflectometry on **Tore Supra** [Clairet RSI 10, Hornung PPCF 13]

- ➥ localised measure, fast (∼ μs), sweeping in X–mode : full radial profile δn
- ➥ **routinely estimate** L_c

Staircase predicted. . .then observed experimentally



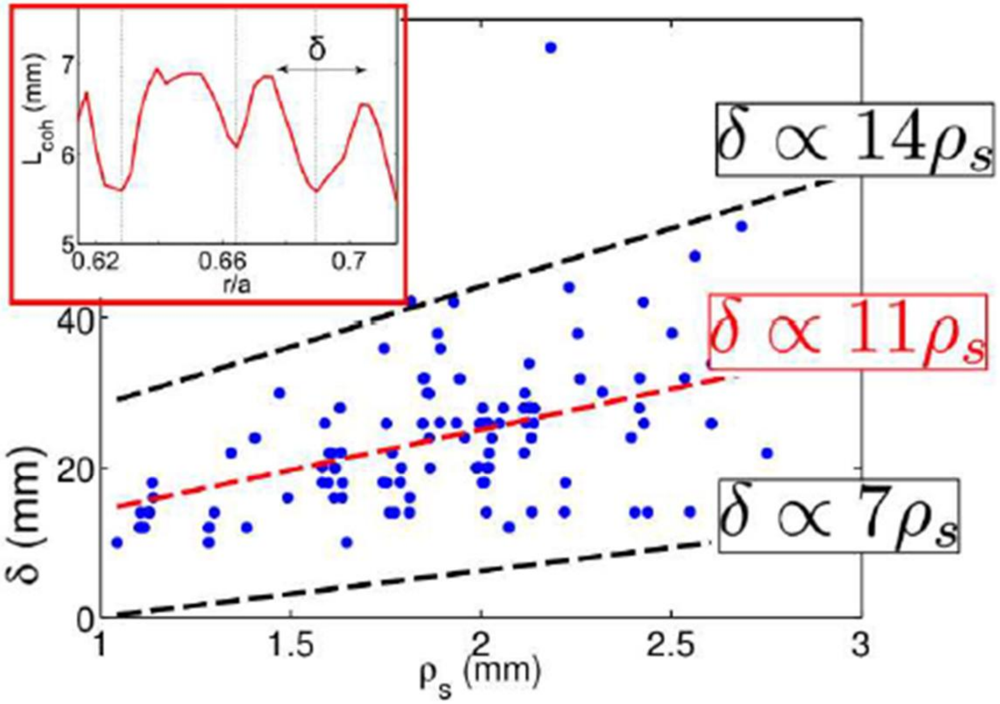


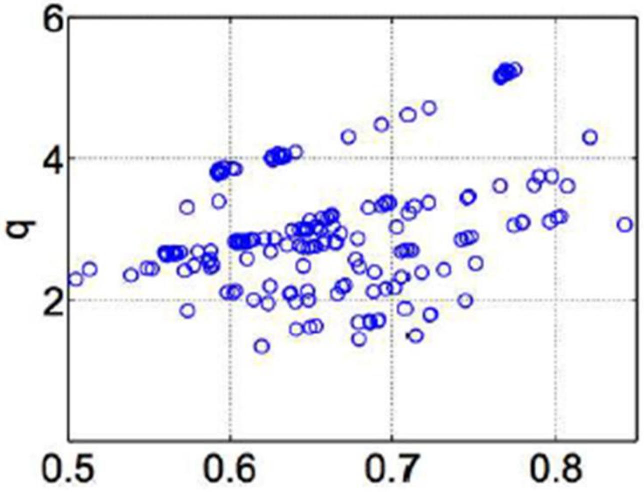
- Large set: 179 staircase steps, so far [Dif-Pradalier PRL 15 & Hornung, in prep.]
  - quasi-regularly spaced radial local minima of L_c
  - reproducible: not random & robust w.r.t. definition of L_c
  - tilt consistent with flow shear around minima

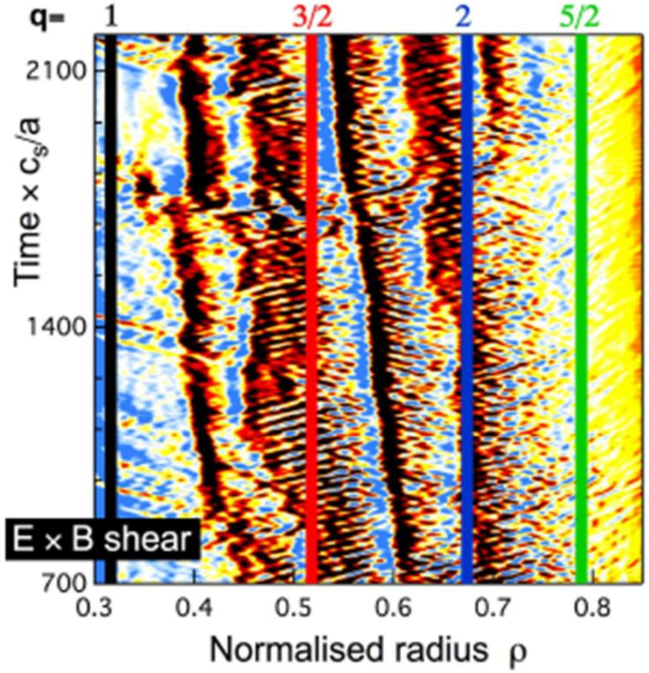
  - no correlation to local q rationals ➠ rules MHD out
  - consistent width $[\sim 10\rho_i]$ & spacing [meso.] of local L_c minima

When **theoretical predictions** lead to **experimental discovery**

- flow width $\delta \sim 11\rho_i$ consistent with Gysela obs. & ZF measurements
  [Fujisawa PRL 04]
- turbulence-borne ➠ not MHD
  [Dif-Pradalier PRL 15 & Hornung, in prep.]

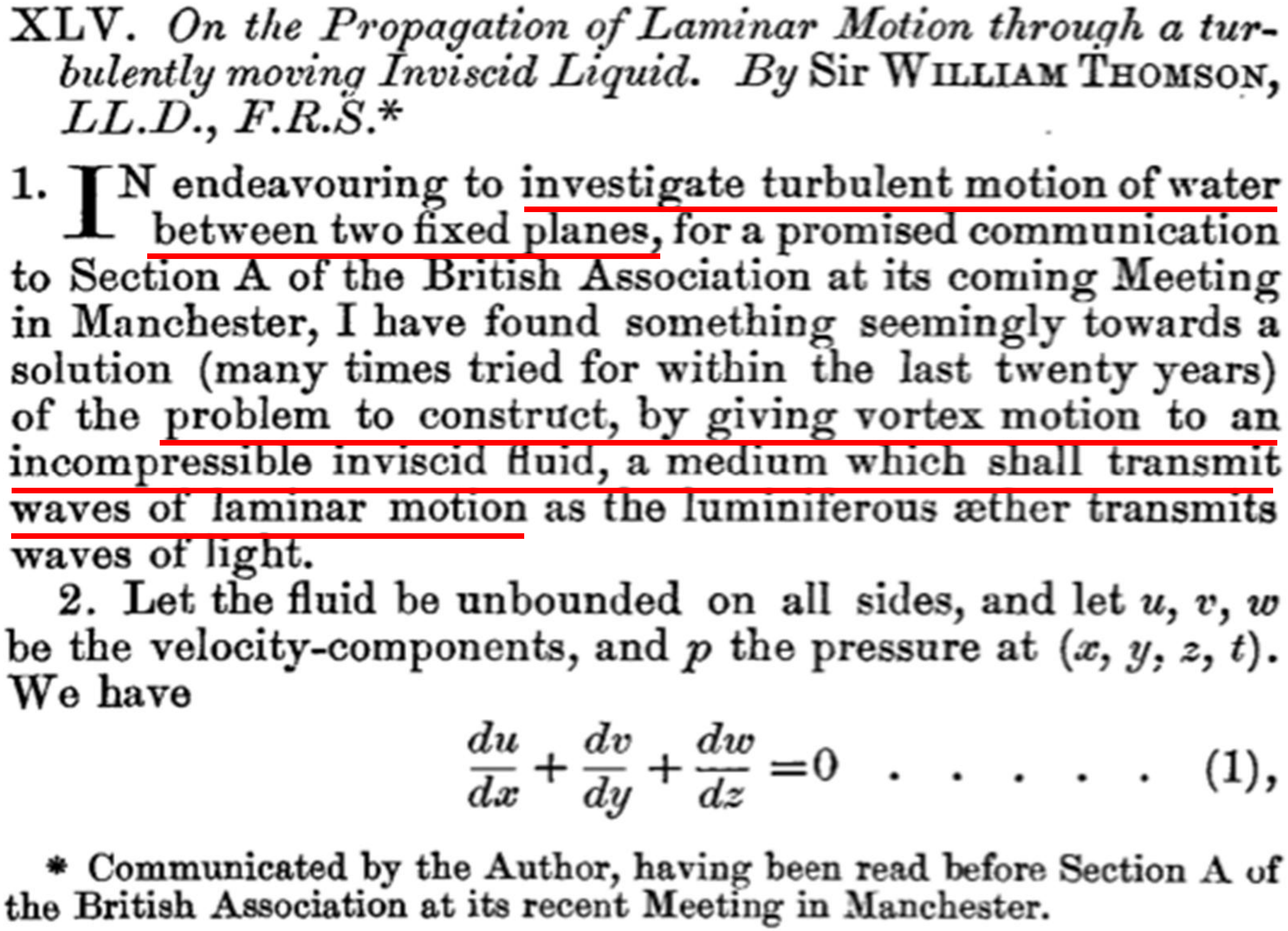






Aside: FYI – Historical Note

- → Collective Dynamics of Turbulent Eddy
- ‘Aether’ I – First Quasi-Particle Model of Transport?!
- Kelvin, 1887



XLV. *On the Propagation of Laminar Motion through a turbulently moving Inviscid Liquid. By* Sir William Thomson, *LL.D., F.R.S.**

1. IN endeavouring to investigate turbulent motion of water between two fixed planes, for a promised communication to Section A of the British Association at its coming Meeting in Manchester, I have found something seemingly towards a solution (many times tried for within the last twenty years) of the problem to construct, by giving vortex motion to an incompressible inviscid fluid, a medium which shall transmit waves of laminar motion as the luminiferous æther transmits waves of light.

2. Let the fluid be unbounded on all sides, and let u, v, w be the velocity-components, and p the pressure at (x, y, z, t). We have

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad (1),$$

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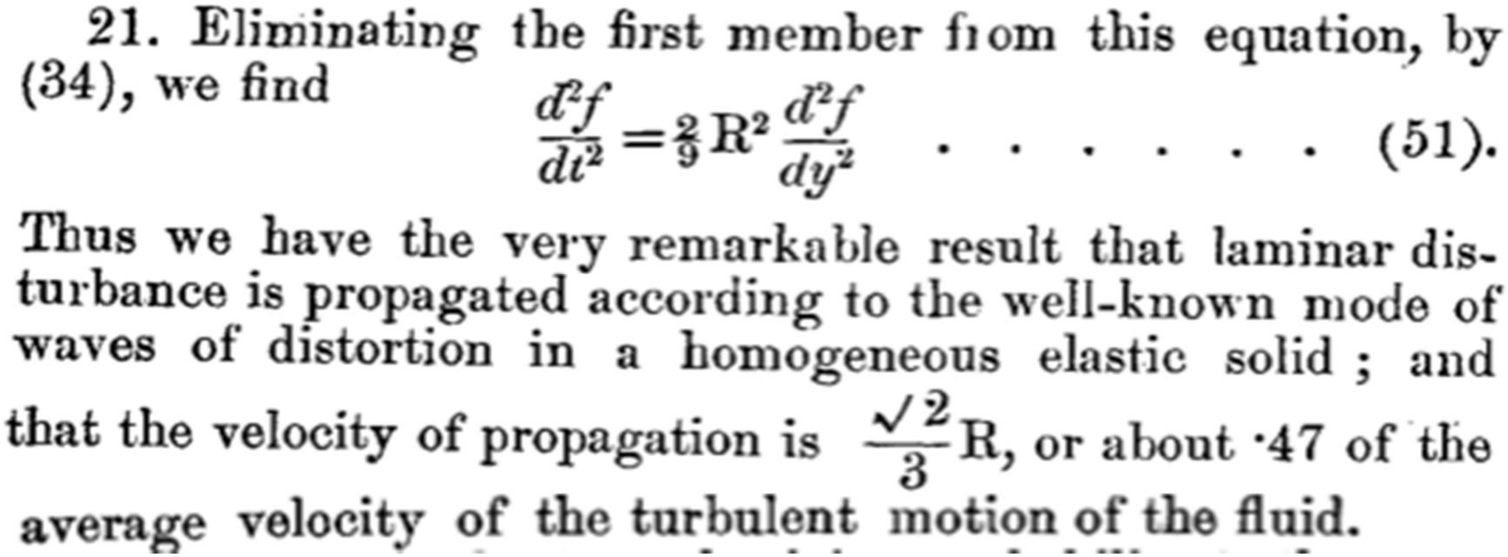
* Communicated by the Author, having been read before Section A of the British Association at its recent Meeting in Manchester.



21. Eliminating the first member from this equation, by (34), we find

$$\frac{d^2f}{dt^2} = \frac{2}{9}R^2\frac{d^2f}{dy^2} \quad . \; . \; . \; . \; . \; . \; . \quad (51).$$

Thus we have the very remarkable result that laminar disturbance is propagated according to the well-known mode of waves of distortion in a homogeneous elastic solid; and that the velocity of propagation is $\frac{\sqrt{2}}{3}$R, or about ·47 of the average velocity of the turbulent motion of the fluid.

$R^2 \sim \langle \tilde{v}^2 \rangle$

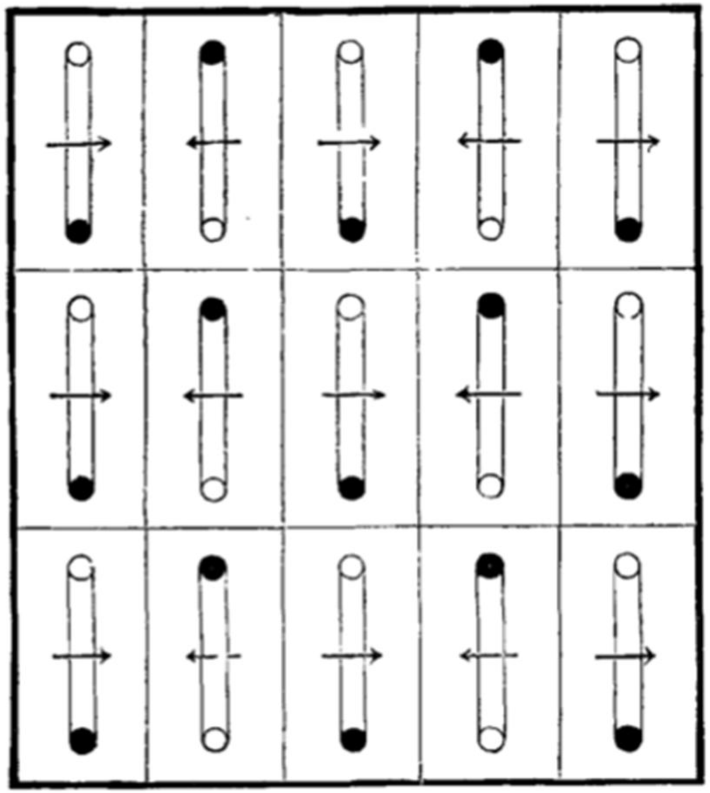
→ time delay between Reynolds stress and wave shear introduced

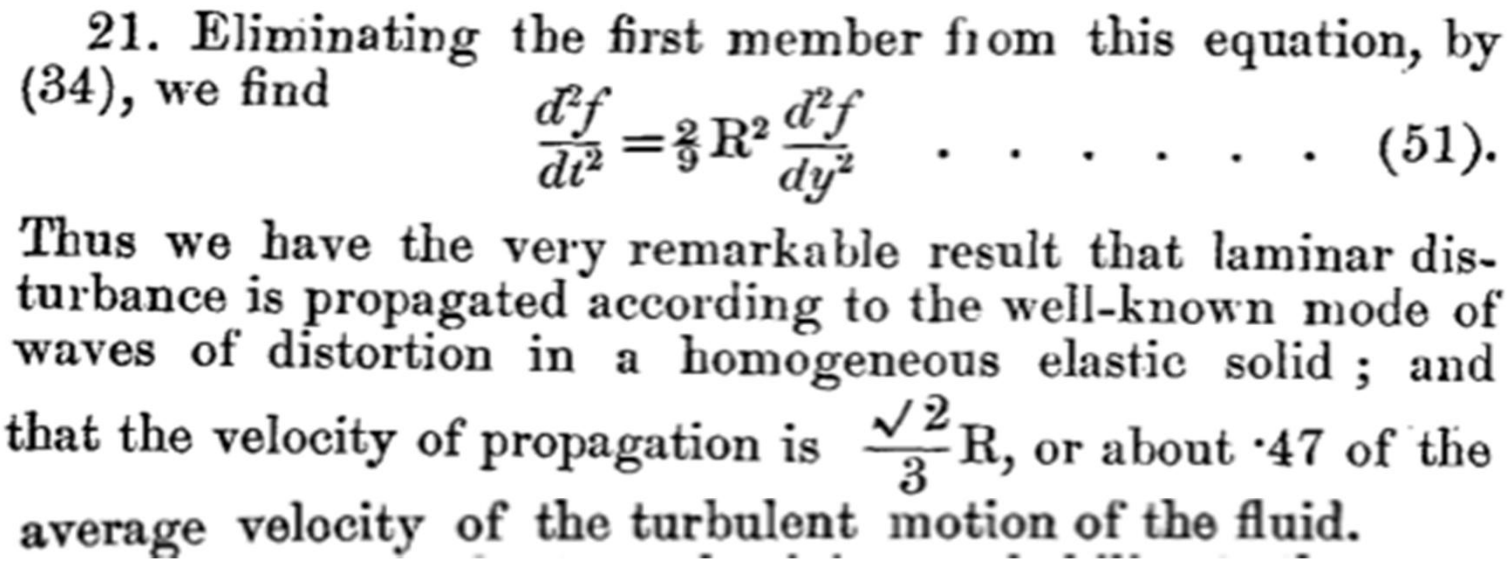
→ converts diffusion equation to wave equation

→ describes wave in ensemble of vortex quasi-particles

– c.f. “Worlds of Flow”, O. Darrigol

Fig. 1.





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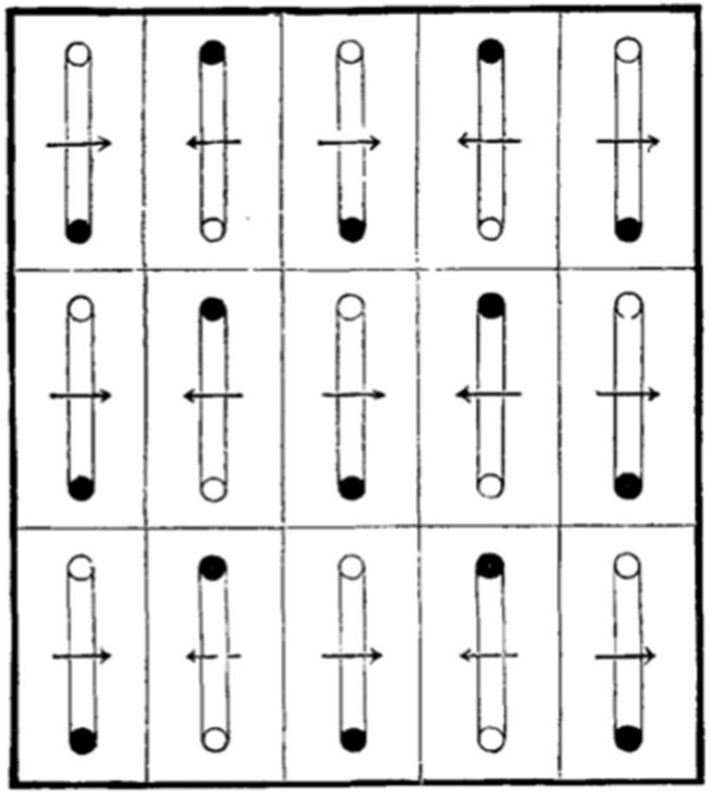
$R^2 \sim \langle \tilde{v}^2 \rangle$

- → time delay between Reynolds stress and wave shear introduced
- → converts diffusion equation to wave equation
- → describes wave in ensemble of vortex quasi-particles
- – c.f. “Worlds of Flow”, O. Darrigol

Fig. 1.



Summary

- A model for ExB staircase formation

  - Heat avalanche jam → profile corrugation → ExB staircase

  - model developed based on analogy to traffic dynamics → telegraph eqn.

- Analysis of heat flux jam dynamics

  - Negative conduction instability as onset of jam formation

  - Growth rate, threshold, scale for maximal growth

  - Qualitative estimate: scale for maximal growth $\Delta \sim 10\Delta_c$

    → comparable to staircase step size

Ongoing Work

- This analysis ↔ set in context of heat transport
- Implications for momentum transport? ➔
  - consider system of flow, wave population, wave momentum flux
  - time delay set by decay of wave population correlation due ray stochastization → elasticity
  - flux limited PV transport allows closure of system

Results:

- Propagating (radially) zonal shear waves predicted, as well as vortex mode
- For τ_{delay} larger, Z.F. state transitions to LCO, rather than fixed point
- τ_{delay} due elastization necessarily impacts dynamics of L→I→H transition

Some Relevant Publications

- Y. Kosuga, P.H. Diamond, O.D. Gurcan; Phys. Rev. Lett. 110, 105002 (2013)
- O.D. Gurcan, P.H. Diamond, et al, Phys. Plasmas 20, 022307 (2013)
- Y. Kosuga, P.H. Diamond, Phys. Plasmas (2014)
- Y. Kosuga: Invited Talk, 2013 APS-DPP Meeting
- Z.B. Guo, P.H. Diamond, et al; Phys. Rev. E, in press (2014)
- Z.B. Guo, P.H. Diamond; Phys. Plasmas (2014)
- G. Dif-Pradalier, Phys. Rev. E (2010, Phys. Rev. Lett. (2015)